

DEFINITIONS

CENTER OF MASS

The center of mass is the point in a body where if all the mass was concentrated at one point, the body would act the same for any direction of linear acceleration. If a force vector passes through this point the body will move in a straight line, with no rotation. Newton's second law of motion describes this motion as $\mathbf{F} = m\mathbf{a}$, where the sum of forces, \mathbf{F} , acting on a body is equal to its mass, m , times its acceleration, \mathbf{a} .

For normal balancing applications, the mass center and the center of gravity occur at the same point. This does not hold true for applications involving a non-uniform gravitational field, however, the scale of most balancing applications is very small with respect to gradients in the earth's gravitational field and the terms are synonymous.

SHAFT AXIS (or GEOMETRIC AXIS)

The shaft axis is also referred to as the geometric axis or the engineered axis of rotation. This axis of rotation is determined either by the rotational bearing surface, which exists on the rotor, or by the mounting surface. An adequate mounting surface establishes the center of rotation at the center of mass plane (the plane in which the center of mass is located). The quality of the mounting datum greatly influences the ability to balance a part. Non-circular surfaces, non-flat surfaces, irregular or loose bearings all allow or cause variations in the position of the rotation axis. Any variation of the axis appears to be motion of the mass center with respect to the axis and contributes to non-repeatability.

PRINCIPAL INERTIA AXIS

When a part is not disc shaped and has length along the axis of rotation, it spins in free space about a line. This line is called the principal inertia axis. The center of mass is a point on this line. It takes energy to disturb a part and cause it to wobble or spin on another inertia axis. Examples of this would be a correctly thrown football or a bullet shot from a rifle.

When the principal inertia axis coincides with the axis of rotation the part will spin with no unbalance forces. In this case, the static as well as the couple unbalance are equal to zero.

In balancing, the term principal inertia axis is used to designate the central principal axis, most nearly coincident with the shaft axis of the rotor.

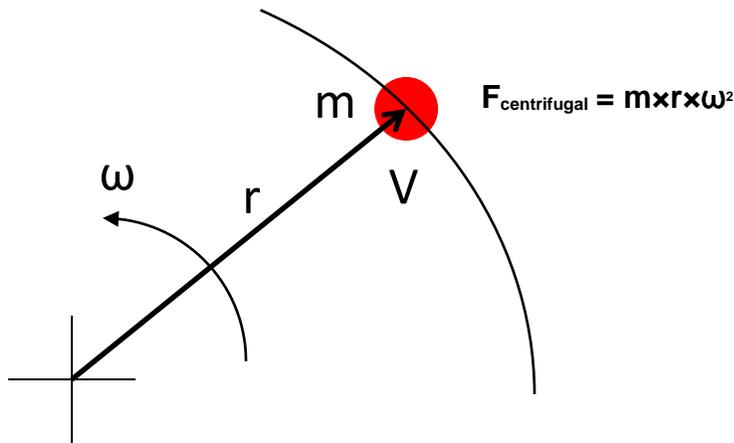
The mass moment of inertia is the rotational counterpart of mass and is a measure of mass distribution about an axis. For a particle it is the product of mass times the square of the distance from the axis to the particle, $I = m \times r^2$. For a rigid body it is an integral, $I = \int r^2 \times dm$.

Since the mass moment of inertia is calculated with respect to an arbitrarily specified axis, it can have just about any value depending on the axis chosen. It turns out that all rigid bodies have at least one set of axes about which the body is perfectly balanced. These axes are known as the principal axes. They are mutually orthogonal and have their origin in the mass center. There are corresponding principal moments of inertia for each.

In balancing, it's useful to describe the central principal axis as the principal axis that is most closely in line with the axis of rotation. It is also known as the *balance axis* or the *mass axis*. A rotor with an axis of rotation that is not coincident with the central principal axis has unbalance. The magnitude of unbalance will be a function of the angle between the axes and the distance of the origin (mass center) from the axis of rotation.

CENTRIFUGAL FORCE

A particle made to travel along a circular path generates a centrifugal force directed *outward* along the radial line from the center of rotation to the particle. As the particle rotates about the center point, so does the centrifugal force.



Centrifugal force is an inertia force and is actually the body's reaction to an externally applied force. For circular motion the external force is known as centripetal force. The centripetal force acts on the particle in a radial *inward* direction. They both have the same magnitude, but differ in the direction of action.

Similarly, a rotor with mass center slightly displaced from the axis of rotation will generate a centrifugal force. This is the force associated with the static unbalance. The shaft supports counteract the forces of unbalance the externally applied centripetal force.

It should be noted here that the quantity $m \times r$ is known as unbalance and that centrifugal force is the product of unbalance and angular velocity squared. While unbalance force ($F_{\text{centrifugal}}$) increases rapidly with speed, the unbalance quantity itself ($m \times r$) does not change at all.

With rigid bodies the unbalance remains the same, although an increase in speed causes an increase in force. The increased force will in turn cause increased motion, depending on the stiffness of the shaft or the shaft supports. Force increases exponentially as the square of the change in speed. Twice the speed equates to four times the force and four times the motion.

WEIGHT AND MASS

The units of weight and mass are often used interchangeably and somewhat loosely in balancing. This is generally acceptable provided the balance computer displays units that are consistent or easily converted to those of the weights in use or the scale used to make the weights. The distinction between weight and mass becomes an issue when calculating unbalance force. It should be understood that weight and force have the same units, Newtons (N). Mass has the units of grams (g) or kilograms (kg).

$$F = m \times r \times \omega^2$$

F, force in Newtons

m, mass in kilograms

r, radius in meters

ω , angular velocity in radians/sec.

TYPES OF UNBALANCE

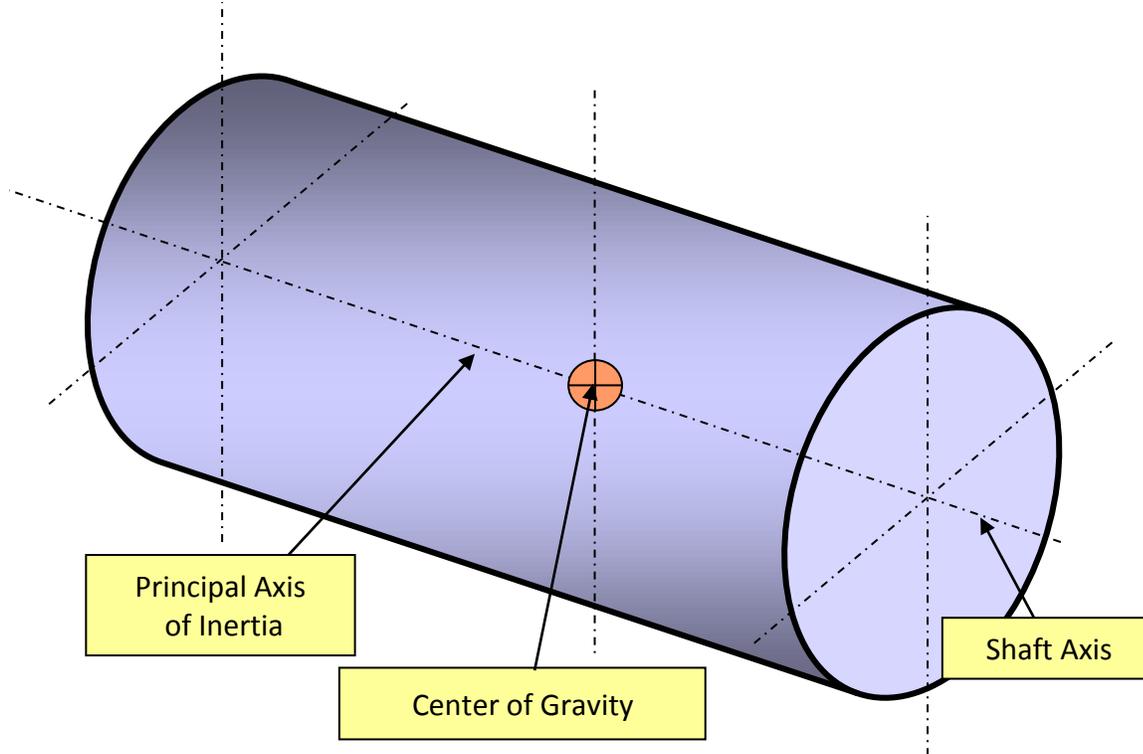
The location of the mass center and the principal inertia axes are determined by the distribution of mass within the part. Unbalance exists when the axis of rotation is not coincident with a principal inertia axis.

It is important to draw a distinction between unbalance and balance correction. Unbalance is a mass property. It becomes a characteristic of the part when an axis of rotation is defined. Balance correction is a means to alter the mass properties to improve the alignment of the axis of rotation with the mass center and/or the central principal axis. Both can be expressed as mass and radii and have shared terminology. However, any condition of unbalance can be corrected by applying or removing weight at a particular radius and angle.

In fact the amount of unbalance, **U**, can be correctly stated as a mass, **m**, at radius, **r**.

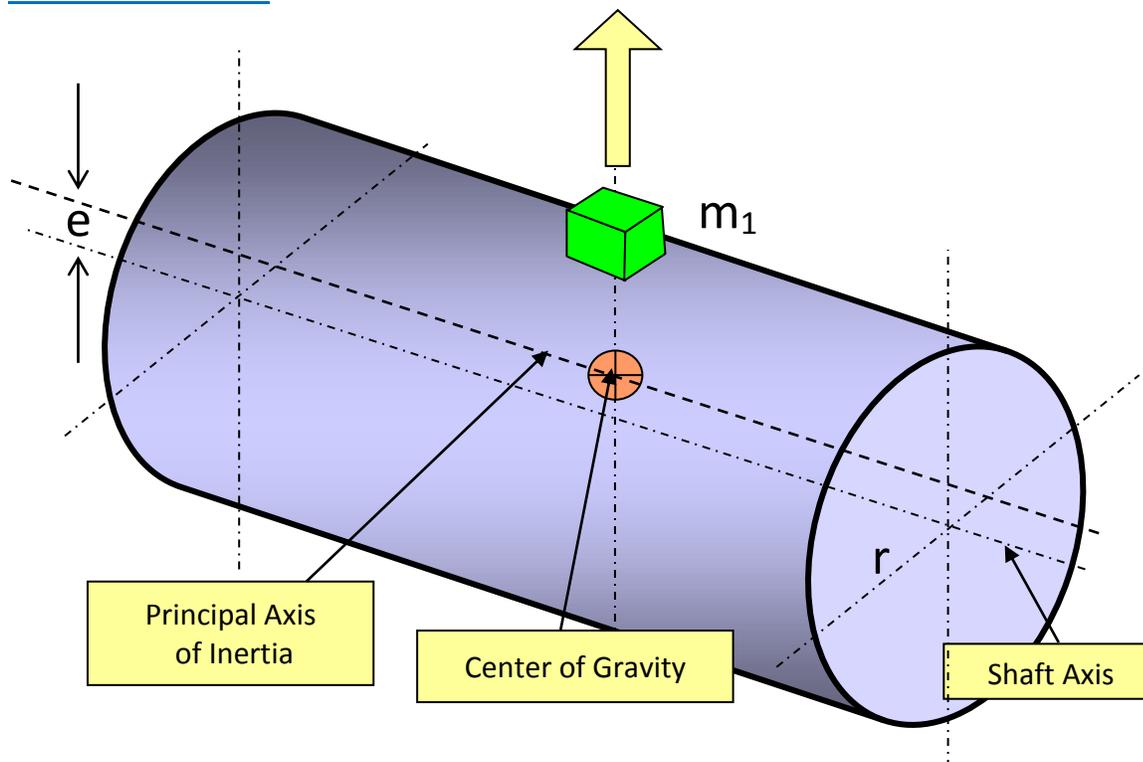
$$U = m \times r$$

BALANCED ROTOR



In this case the shaft axis is identically with the Principal Axis of Inertia.

STATIC UNBALANCE

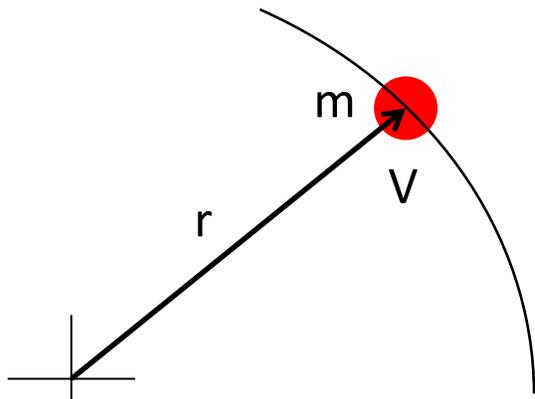


e = mass center displacement

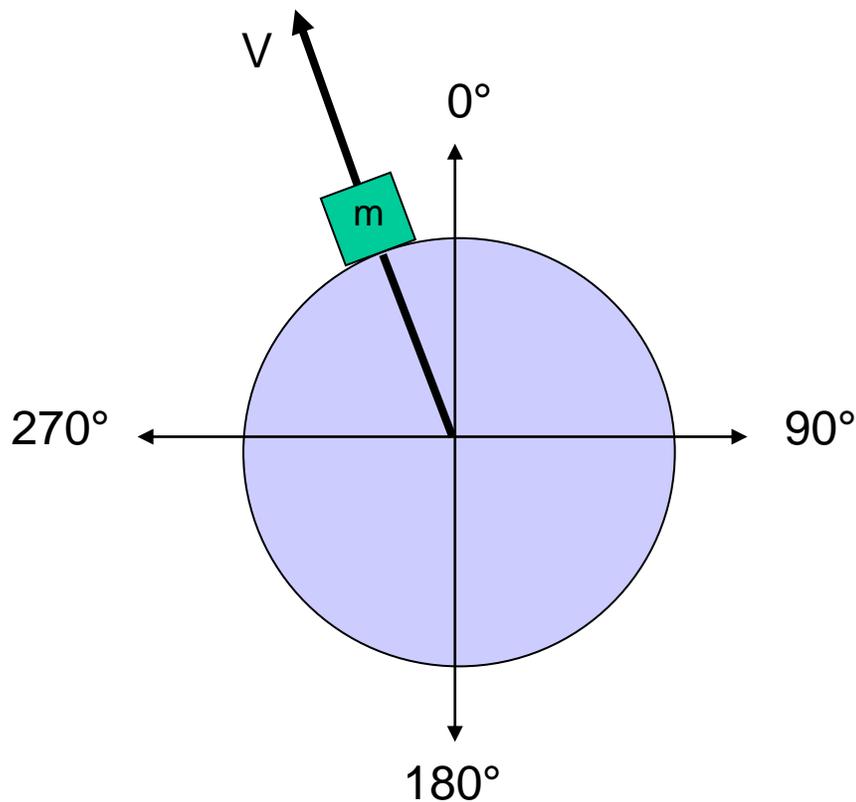
STATIC UNBALANCE - that condition of unbalance for which the central principal axis is displaced only parallel to the shaft axis.

It's a condition that exists when the center of mass is not on the rotation axis. Static unbalance by itself is typically measured and corrected on narrow disc-shaped parts. To correct the static unbalance requires only one correction. The amount of unbalance is the product of the mass and radius. This type of unbalance is a vector, and therefore, must be corrected with a known mass at a particular angle.

As defined, static unbalance is an ideal condition, it has the additional condition that the axis of rotation be parallel to the central principal axis, no couple unbalance.



m : unbalance mass
 r : distance from center (radius)
 V : unbalance vector

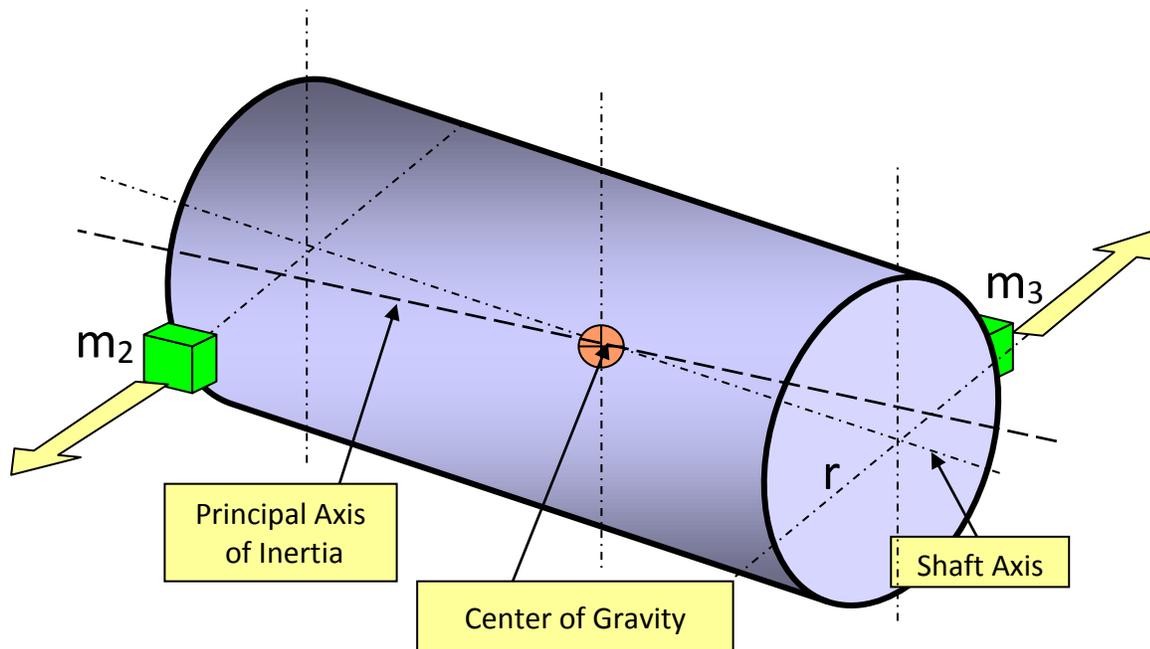


(Vector - a quantity possessing both magnitude and direction)

Static unbalance can be determined if you know the unbalance mass and the displacement of the mass center from the geometric axis.

As discussed earlier, a rotor is in static balance when the center of mass is on the rotation axis. When this condition exists, the part can spin on this axis without creating inertial force on the center of mass.

COUPLE UNBALANCE



COUPLE UNBALANCE - that condition of unbalance for which the central principal axis intersects the shaft axis at the center of gravity.

A couple is a system of two parallel forces, equal in magnitude and acting in opposite directions. A couple causes a moment or torque proportional to the distance between the parallel forces. Its effect is to cause a twisting or turning motion.

It is a specific condition that exists when the principal inertia axis is not parallel with the axis of rotation. To correct the couple unbalance, two equal weights must be added to the rotor at angles 180° apart in two correction planes. The distance between these planes is called the couple arm. Couple unbalance consists of two vectors that describe the correction.

Couple unbalance can be corrected in any two planes, but first the amount must be divided by the distance between the chosen planes.

Couple unbalance appears as the off-diagonal terms in the inertia matrix for a rigid body.

This is an indication that the inertial axes are not aligned with the principal axes. It can be expressed as a vector with direction perpendicular to the plane of the radius vector and the couple arm vector.

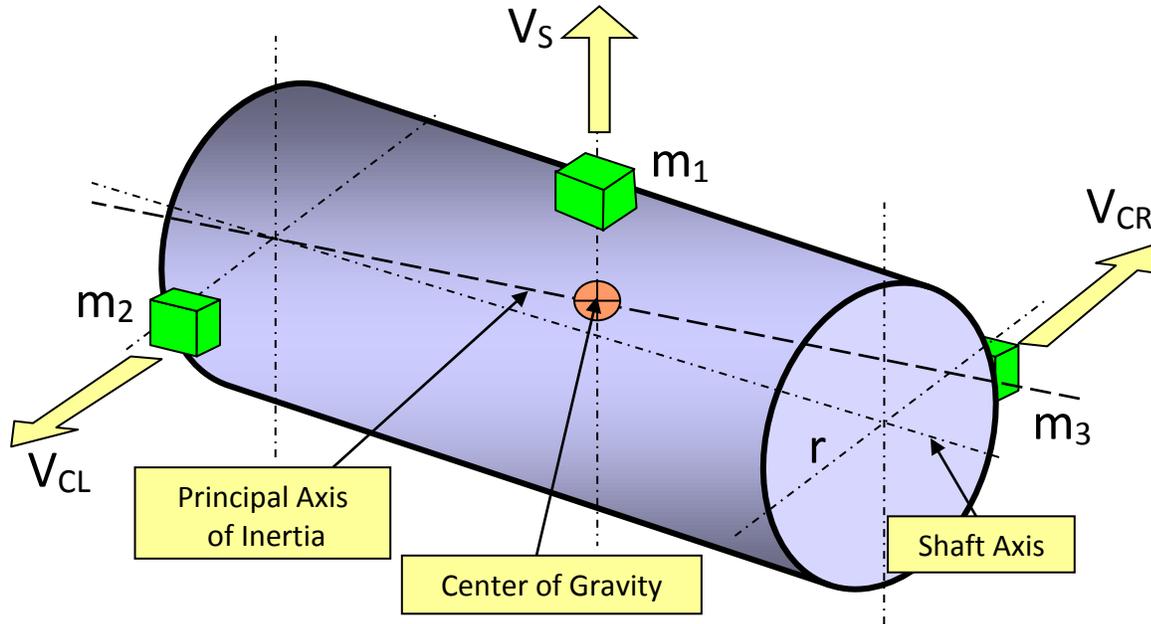
Whereas static unbalance can be measured with a non-rotating balancer, couple unbalance can only be measured on a rotating balancer.

A combination of static and couple unbalance fully specifies all the unbalance which exists in a rotor. Specifying unbalance in this manner requires three individual correction mass.

DYNAMIC UNBALANCE

DYNAMIC UNBALANCE - that condition of unbalance for which the central principal axis is not parallel to and does not intersect the shaft axis. It may also be defined as a combination of static and couple unbalance at different angles.

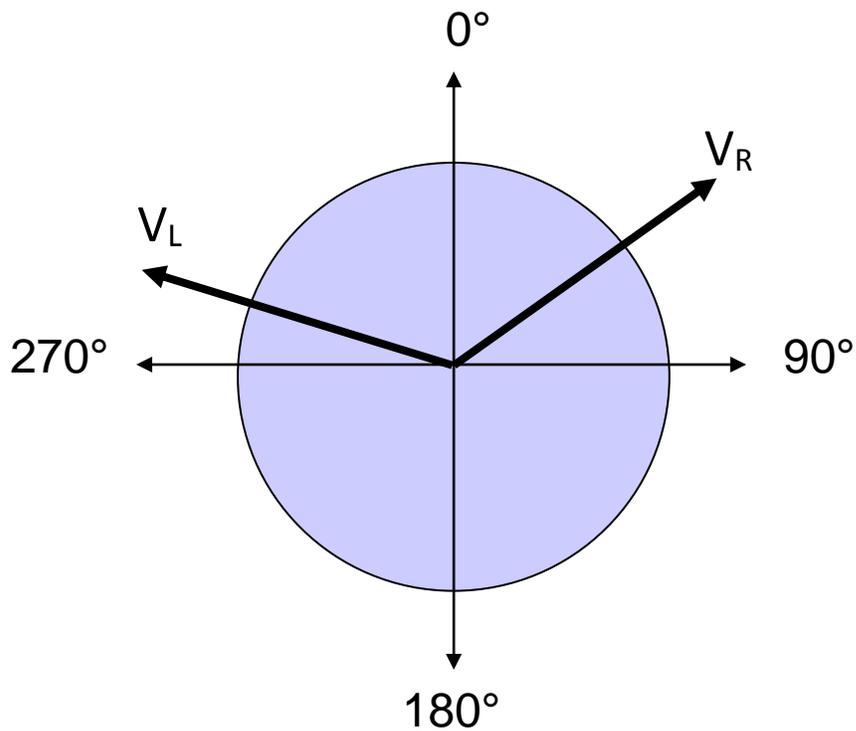
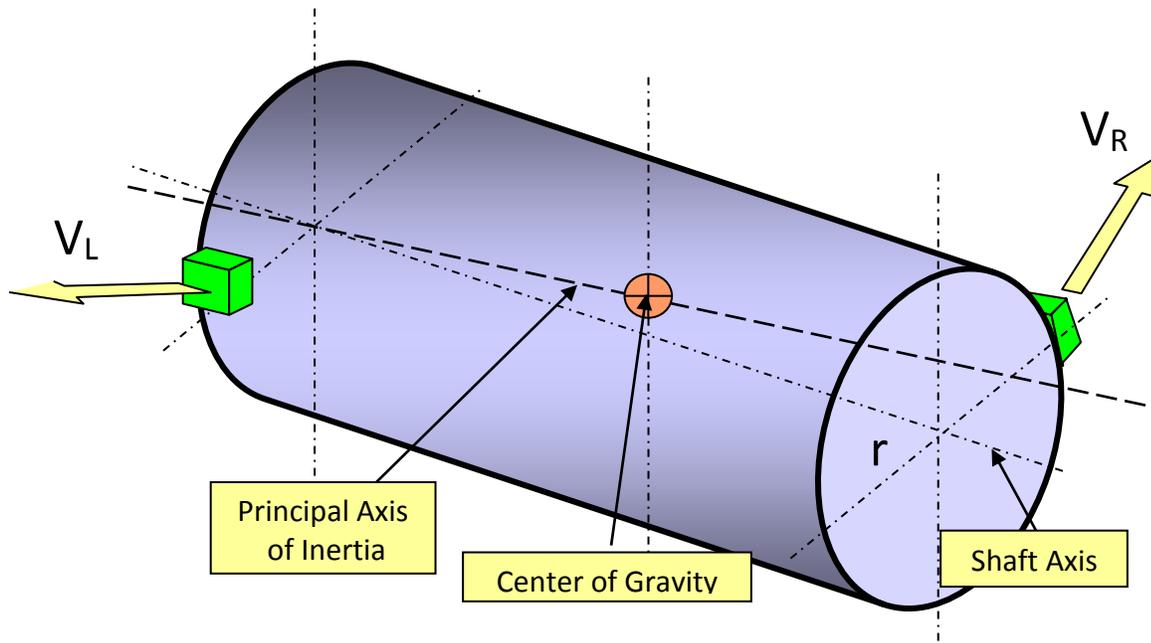
Dynamic Unbalance (Static and Couple unbalance vectors)



Principal Axis of Inertia does **not** intersect with shaft axis.

$$V_{CL} = V_{CR}$$

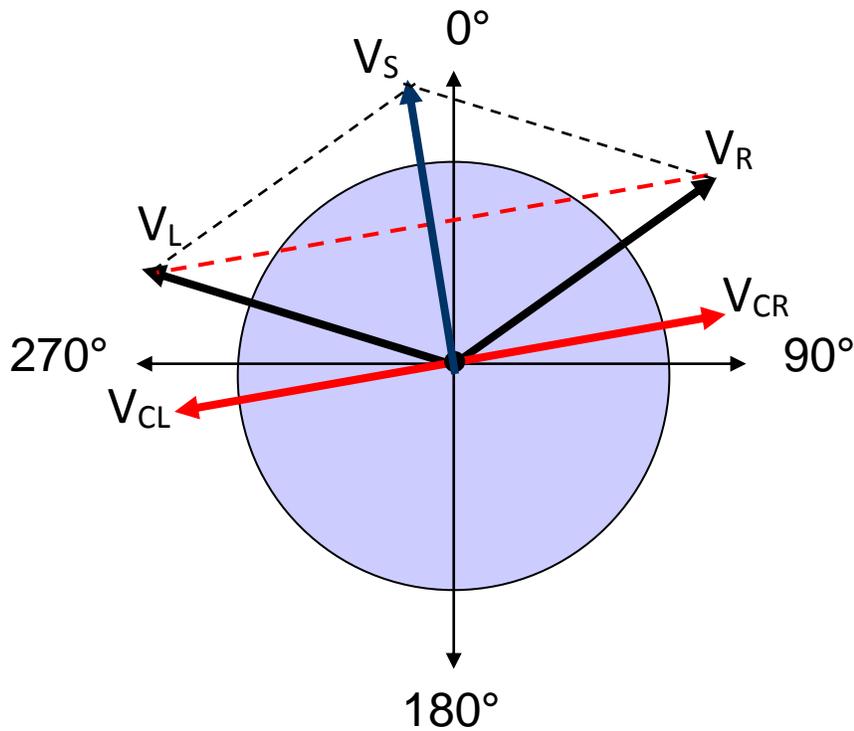
Dynamic Unbalance (Left and Right unbalance vectors)



Dynamic Unbalance (Static and Couple unbalance vectors)

Left and Right unbalance can be divided into static and couple unbalance equivalents. This can be done graphically by plotting the two plane unbalance vectors V_L and V_R as shown bellow.

Connect vectors V_L and V_R as shown. The vector from the origin to the *mid-point* of vector $V_L - V_R$ is one-half the rotor's static unbalance, V_S . Vectors V_{CL} and V_{CR} are the couple unbalance.



Correction planes

Rotors out of balance tolerance need correction. These unbalance corrections often cannot be performed in the planes where balance tolerances were set, but need to be performed where material can be added, removed or relocated.

The number of necessary correction planes depends on the magnitude and distribution of the initial unbalance as well as on the design of the rotor, for instance the shape of the correction planes and their location relative to the tolerance planes.

Rotors which need one correction plane only

For some rotors, only the static unbalance is out of tolerance, the couple unbalance is in tolerance. This typically happens with disc-shaped rotors, provided that

- the bearing distance is sufficiently large,
- the disc rotates with sufficiently small axial run out,
- the correction plane for the static unbalance is properly chosen;

Whether these conditions are fulfilled, each individual case can be investigated. After single-plane balancing has been carried out on a sufficient number of rotors, the largest residual couple unbalance is determined and divided by the bearing distance, yielding a couple unbalance (pair of unbalances). If, even in the worst case, the unbalances found this way are acceptable, it can be expected that a single-plane balancing is sufficient.

For single-plane balancing, the rotor needs not rotate, but for sensitivity and accuracy reasons in most cases rotational balancing machines are used. The resultant unbalance can be determined and corrected to limits.

Rotors which need two correction planes

If a rotor in a constant (rigid) state does not comply with the conditions as stated in above, the couple unbalance needs to be reduced as well. In most cases, static unbalance and couple unbalance are assembled into a dynamic unbalance: two unbalance vectors in two planes, called complementary unbalance vectors.

For two-plane balancing, it is necessary for the rotor to rotate, since otherwise the couple unbalance would remain undetected.

Rotors with more than two correction planes

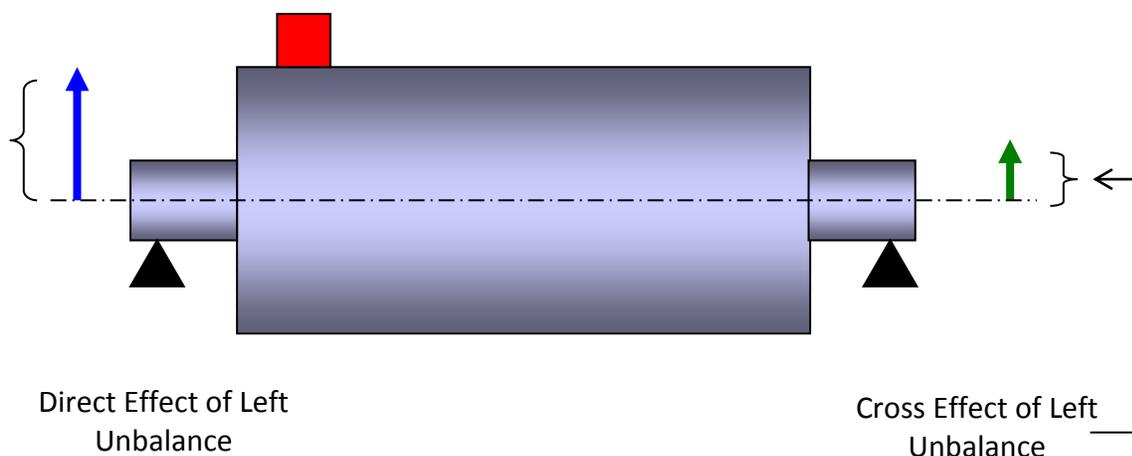
Although all rotors in their constant (rigid) state theoretically can be balanced in two planes, sometimes more than two correction planes are used, for instance:

- in case of separate corrections of static unbalance and couple unbalance, if the correction of the static unbalance is not performed in one (or both) of the couple planes
- if the correction is spread along the rotor.

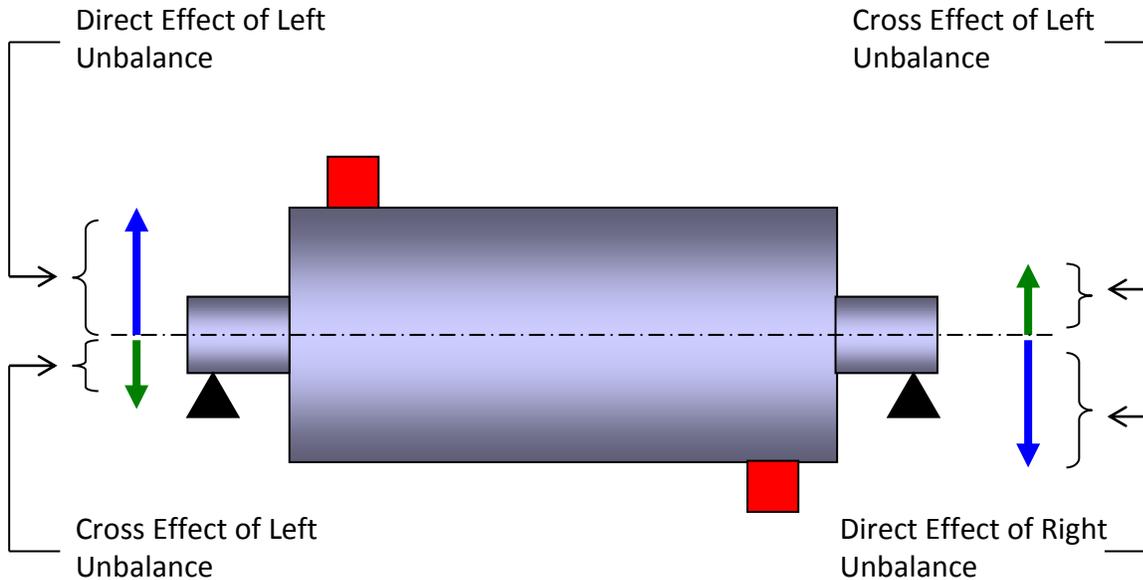
Plane Separation

Static Unbalance

Unbalance mass at left end of the rotor causes left end to vibrate, but also has a cross effect on the right end.



Couple Unbalance



Unbalance Reduction Ratio

- Measures overall efficiency of unbalance correction.
- Ratio of the reduction in the unbalance by a single balancing correction to the initial unbalance.
- Usually given as a percentage.

$$URR = \frac{U_1 - U_2}{U_1} = 1 - \frac{U_2}{U_1}$$

U_1 : the amount of initial unbalance

U_2 : the amount of residual unbalance after performing one correction.

RIGHT-LEFT CORRECTION

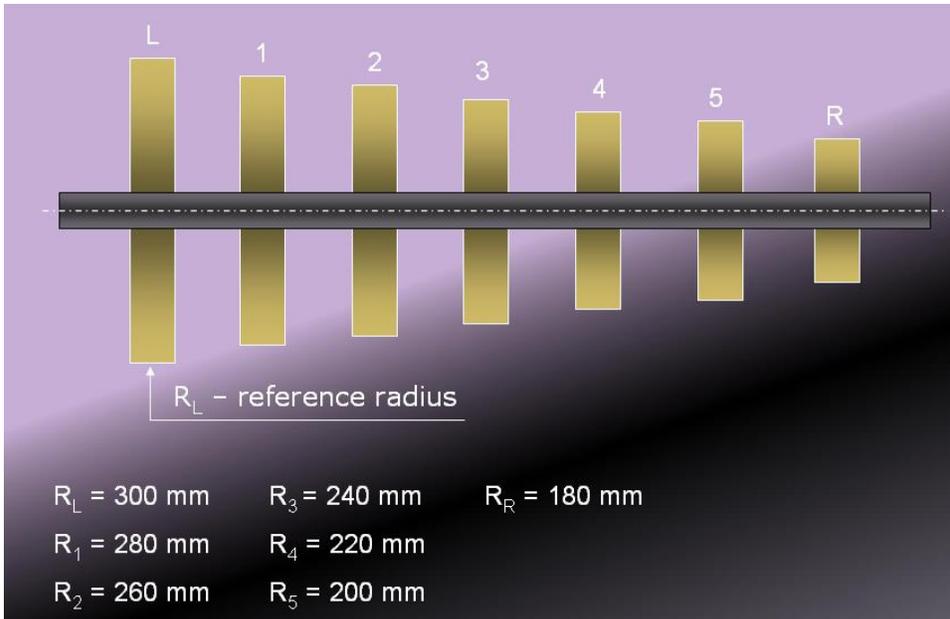
Right-Left correction is a two step process. Two balance corrections are made in predefined left and right planes. The CXBalancer calculates and displays four values: both amount and angle for the left plane and amount and angle for the right plane.

STATIC-COUPLE CORRECTION

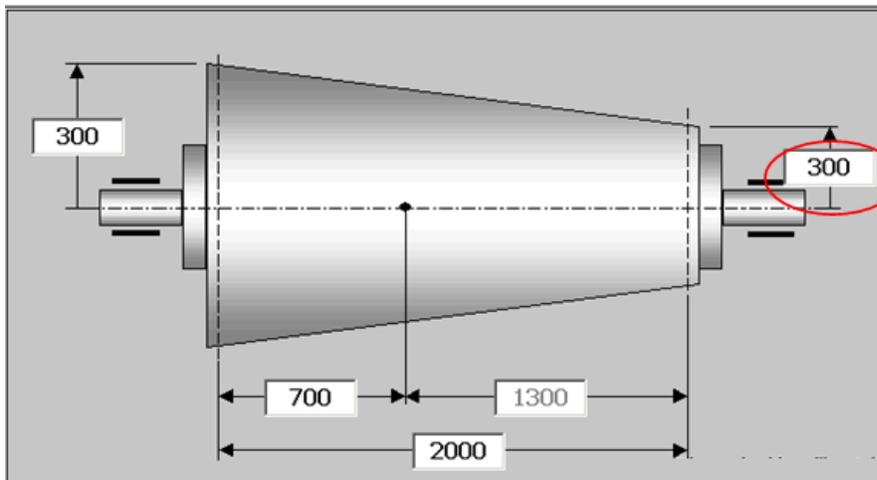
Static-Couple correction is a four step process. Four balance corrections are made in predefined left and right planes. The CXBalancer calculates and displays six values; amount and angle for a static correction and amount and angle for a couple correction. The static correction should be divided by two and applied at the same angle in both the left and right planes or spread along the rotor. The couple correction should be done in the left and right planes at the specified angle.

PRACTICAL EXAMPLE

Assuming that we have to balance a multi-stage pump rotor:



First we must enter the same radius for the most left and most right rotors:



In the second step we must select an arbitrary reference radius (in example above, we will select the radius of the left rotor - R_L).

Then, we will perform the normal two-plane balancing algorithm.

By adding a trial weight of 35 grams, the unbalance calculation result will be as below:

LEFT BALANCING PLANE		RIGHT BALANCING PLANE	
AMOUNT (gr.)	ANGLE(°)	AMOUNT (gr.)	ANGLE(°)
2.69	188.8	3.77	312.0

Now, we must determine the amount and the angle for static and couple unbalance.

Just pressing the MENU key and selecting STATIC/COUPLE, the instrument will show the following results:

STATIC UNBALANCE		COUPLE UNBALANCE			
AMOUNT (gr.)	ANGLE(°)	LEFT		RIGHT	
3.21	267.6	2.80 gr.	175.3°	2.80 gr.	355.3°

Observe that the amount of the couple unbalance is equal, but the angle difference is exactly 180°.

The couple unbalance will be corrected on to the most LEFT and most RIGHT rotors (R_L and R_R). The static unbalance will be spited between mid rotors. In our case, the correction weight for each intermediate rotor will be 0.642 grams.

The instrument calculates the correction weight considering the reference radius, but this is not true, except for the selected reference radius (R_L).

So, we must re-calculate the real correction weight for each rotor.

This can be easy done, considering that the product mass - radius has to be constant:

ROTOR	REFERENCE RADIUS (mm)	ACTUAL RADIUS (mm)	CORRECTION WEIGHT (gram)	
			From result	Recalculated
L	300	300	2.800	2.800
1	300	280	0.642	0.688
2	300	260	0.642	0.741
3	300	240	0.642	0.802
4	300	220	0.642	0.875
5	300	200	0.642	0.963
R	300	180	2.800	4.770

Now we must prepare seven correction weights and we have to place them as follows:

ROTOR	ACTUAL RADIUS (mm)	CORRECTION WEIGHT (gram)	
		Amount (gr.)	Angle(°)
L	300	2.800	175.3
1	280	0.688	267.6
2	260	0.741	267.6
3	240	0.802	267.6
4	220	0.875	267.6
5	200	0.963	267.6
R	180	4.770	355.3

Notice that the static unbalance is corrected on the same angle and the couple unbalance is corrected with two weights placed at 180° apart.

The result after correction is shown below:

	LEFT BALANCING PLANE		RIGHT BALANCING PLANE	
	AMOUNT (gr.)	ANGLE (°)	AMOUNT (gr.)	ANGLE(°)
Initial run	2.69	188.8	3.77	312.0
After correction	0.896	218.1	0.315	339.4

If the rotor is still out of tolerance, a small weight correction can be done as usual, in the LEFT and RIGHT plane only.

This correction method is widely used on disk shaped rotors, or when provisions for weight correction are made in multiple planes. The benefits of using this method to correct the static unbalance allows a better approximation of rotor unbalance condition, avoids making excessive corrections in one or two planes and helps when a rotor operates near its critical bending mode (1st free- free mode).

IMPORTANT NOTE

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